# Petri nets: Modeling and Applications

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# Outline

- Definition of Petri nets
- Petri net system
- Reachability graph
- Petri net models
- Timed Petri nets
- Applications

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# Definitions

Petri Nets (PN) offer advantages because of their twofold representation: graphical and mathematical.

It is a graphical and mathematical tool for the formal description of the logical interactions and the dynamics of complex systems. PN are particularly suited to model:

- Concurrency
- Conflict
- Sequencing
- Synchronization
- Sharing of limited resources
- Mutual exclusion

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The main International Conference: ATPN - Application and Theory of PN Other conferences: Int. IFAC conference, IEEE SMC conference, IEEE CASE, WODES, IEEE CDC etc.

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A Petri net is a bipartite directed graph consisting of two kinds of nodes: places and transitions

Places typically represent conditions within the system being modeled. They are represented by circles.

Transitions represent events occurring in the system that may cause change in the condition of the system. They are represented by bars.

Arcs connect places to transitions and transitions to places (never an arc from a place to a place or from a transition to a transition).

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Input arcs are directed arcs drawn from places to transitions, representing the conditions that need to be satisfied for the event to be activated

Output arcs are directed arcs drawn from transitions to places, representing the conditions resulting from the occurrence of the event.

Input places of a transition are the set of places that are connected to the transition through input arcs.

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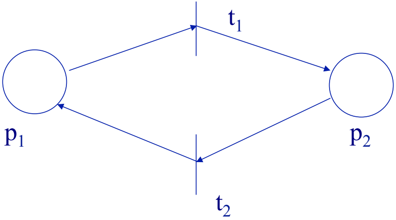



$p_1$ – resource idle

$p_2$ – resource busy

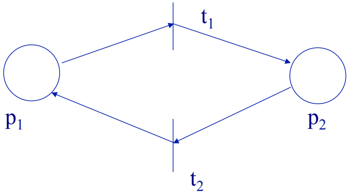
$t_1$ – task arrives

$t_2$ – task completes

# Definitions

Formally, a Petri net is described by the four-tuple
$PN = (P, T, Pre, Post)$
where:
$P$ is the set of places and $|P| = m$
$T$ is the set of transitions and $|T| = n$
$Pre : P \times T \rightarrow N$ is the pre incidence matrix, that specifies the arcs directed from places to transitions
$Post : P \times T \rightarrow N$ is the post incidence matrix, that specifies the arcs directed from transitions to places

# Example




$p_1$ – resource idle
$p_2$ – resource busy
$t_1$ – task arrives
$t_2$ – task completes

$P = \{p_1, p_2\}$, $|P| = 2$ set of places
$T = \{t_1, t_2\}$, $|T| = 2$ set of transitions

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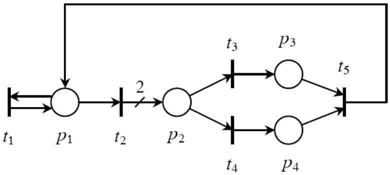
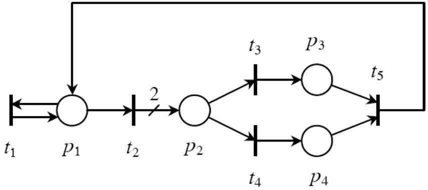



$$P = \{p_1, p_2, p_3, p_4\}, \quad T = \{t_1, t_2, t_3, t_4, t_5\}$$

$$Pre = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{matrix} \qquad Post = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{matrix}$$

$$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \qquad\qquad t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5$$

# Incidence matrix

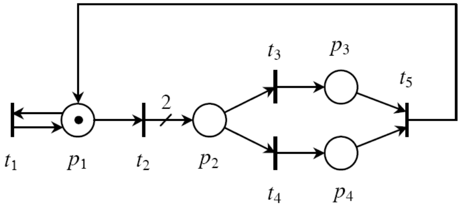



The incidence matrix is $C : P \times T \rightarrow \mathbb{Z}$ is defined as $C = Post - Pre$.

$$C = \begin{bmatrix} 0 & -1 & 0 & 0 & 1 \\ 0 & 2 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{matrix}$$

$$\quad t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5$$

# Marking




Tokens are dots (or integers) associated with places

A place containing tokens indicates that the corresponding condition holds

Marking is a function $M : P \rightarrow N$ that associates to each place a non negative number.

It is a vector listing the number of tokens in each place of the net.

$M_0 = [M_0(p_1) M_0(p_2) M_0(p_3) M_0(p_4)]^T = [1000]^T$

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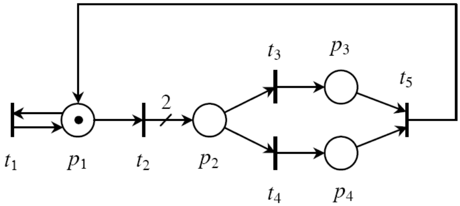


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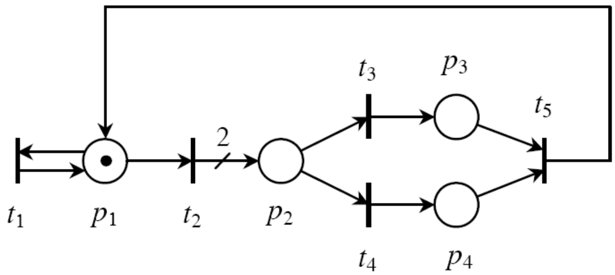
# Net System



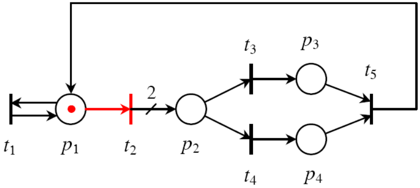

$M_0$ is the initial marking
$PN = (P, T, Pre, Post)$ is a Petri net
The net system $\langle PN, M_0 \rangle$ is a dynamical system

# Rules of Evolution: enabling condition

A transition $t_j \in T$ is enabled at a marking $M$ if and only if for each $p \in \bullet t_j$, it holds $M(p) \geq Pre(p, t_j)$.

Symbol $M[t_j\rangle$ denotes that $t_j \in T$ is enabled at marking $M$.
If $M$ does not enable $t_j$, it holds $\neg M[t_j\rangle$.

# Rules of Evolution: enabling condition




$M_0 \geq Pre(\cdot, t_2)$ then transition $t_2$ is enabled at $M_0$

An enabled transition may fire (event happens) removing one or more tokens from each input place and depositing one or more tokens in each of its output place.

# Rules of Evolution: state equation

When an enabled transition $t_j$ fires, it produces a new marking $M'$:

$$M' = M + Post(\cdot, t_j) - Pre(\cdot, t_j) = M + C(\cdot, t_j)$$

or

$$M' = M + C \cdot \overrightarrow{t}_j$$

where $\overrightarrow{t}_j$ is the $n$-dimensional firing vector corresponding to the $j$-th canonical basis vector.

Symbol $M[t_j\rangle M'$ denotes that $t_j \in T$ is enabled, fires and produces marking $M'$.

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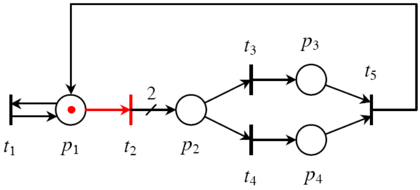


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$t_2$ is enabled and fires...

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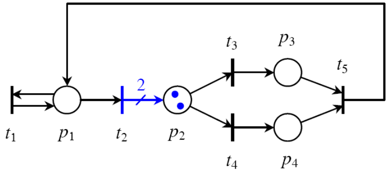


Transition $t_2$ fires yielding a new marking $M_0[t_2 > M$ with

$$M = M_0 - Pre(\cdot, t_2) + Post(\cdot, t_2) = M_0 + C(\cdot, t_2)$$

$$\begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

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# Motivation

A marked net $\langle N, M_0 \rangle$ with $N = (P, T, Pre, Post)$ specifies:

- an initial marking (i.e., state) $M_0$;
- the rules of evolution.

No explicit enumeration of:

- net language, i.e., the set of sequences of transitions that can fire:

$$L(N, M_0) = \{\sigma \in T^* \mid M_0[\sigma\rangle\};$$

- reachability set, i.e., the set of reachable markings:

$$R(N, M_0) = \{M \in \mathbb{N}^{|P|} \mid (\exists \sigma \in L(N, M_0))\ M_0[\sigma\rangle M\}.$$

The information on reachable markings and firing sequences is useful to determine if the net has given properties.

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# Boundedness

**Definition**

A place $p \in P$ is $k$-bounded if for any marking $M \in R(N, M_0)$ it holds $M(p) \leq k$, i.e., in all reachable markings the number of tokens it contains never exceeds $k$.

Useful to determine maximal capacity or overflow of buffers.

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A marked net $\langle N, M_0 \rangle$ is $k$-bounded if all its places are $k$-bounded.

A bounded net has a finite reachability set, while an unbounded net has an infinite reachability set.

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A transition $t \in T$ is quasi-live if there exists a firing sequence $\sigma \in T^*$ such that $M_0[\sigma t\rangle$, i.e., transition $t$ can *eventually* fire.

A transition $t \in T$ is live if for any marking $M \in R(N, M_0)$ there exists a firing sequence $\sigma \in T^*$ such that $M[\sigma t\rangle$, i.e., from from any reachable marking $t$ can *eventually* fire.

Useful to characterize an event that can occur at least once (quasi-liveness) or that can always eventually occur (liveness).

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# Reachability graph

The reachability graph of a marked net $\langle N, M_0 \rangle$ is an automaton $\mathcal{G} = (X, E, \delta, x_0)$ where:

- $X = R(N, M_0)$, i.e., the states of the automaton are the reachable markings;
- $E = T$, i.e., the events in the alphabet are the transitions of the net;
- for any two reachable markings $M, M'$:

$$\delta(M, t) = M' \quad \Longleftrightarrow \quad M[t\rangle M',$$

  i.e., there exists arc labeled $t$ from $M$ to $M'$ on the automaton iff marking $M'$ is reachable from $M$ firing transition $t$;
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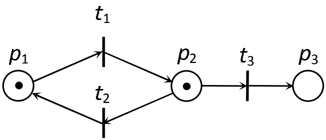


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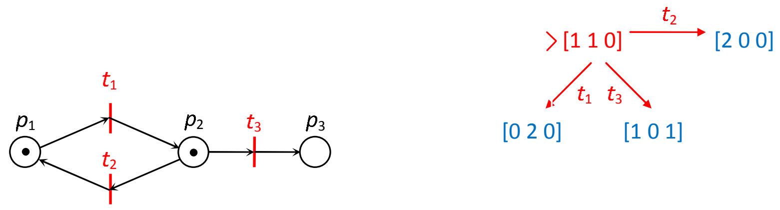

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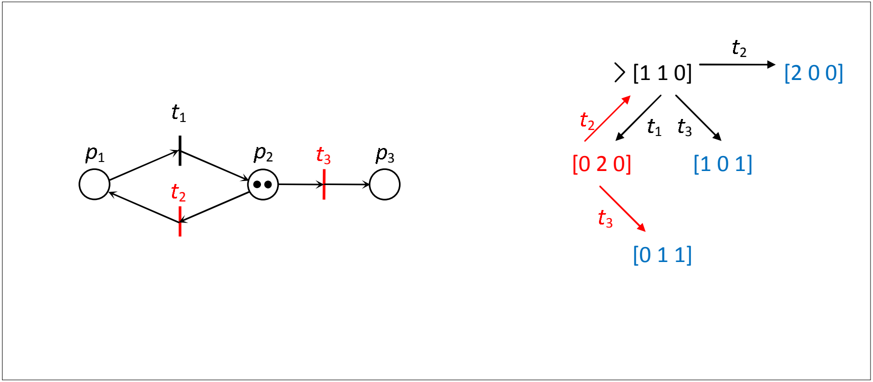

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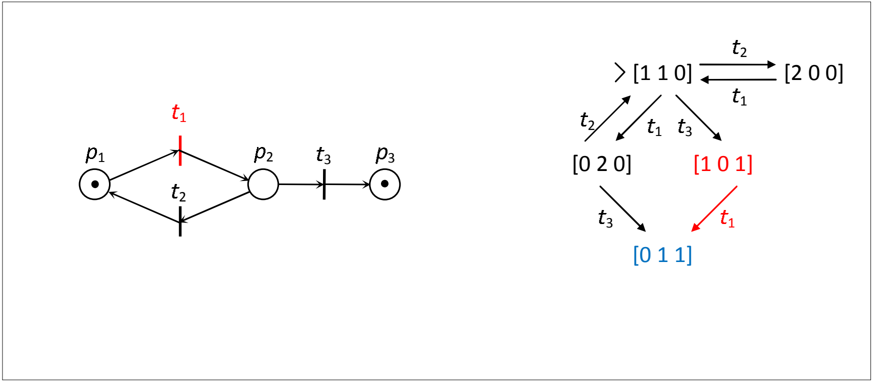

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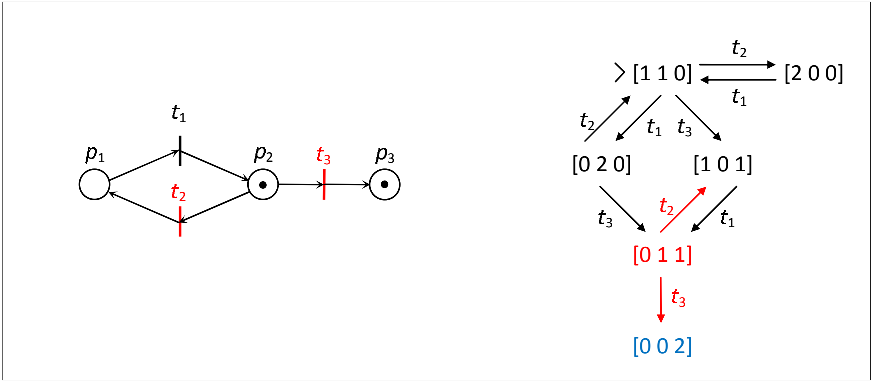
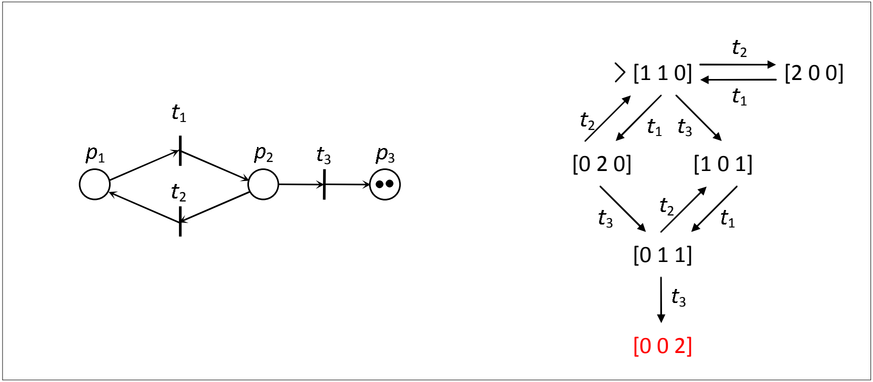
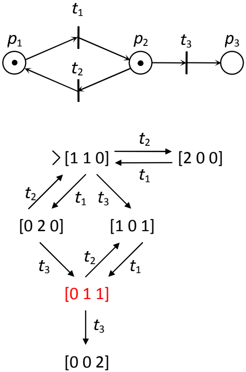


# Example



# Example




Example:

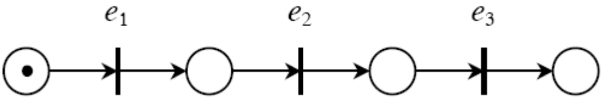
- $M = [0\ 1\ 1]^T$ is reachable

# Outline

- Definition of Petri nets
- Petri net system
- Reachability graph
- **Petri net models**
- Timed Petri nets
- Applications

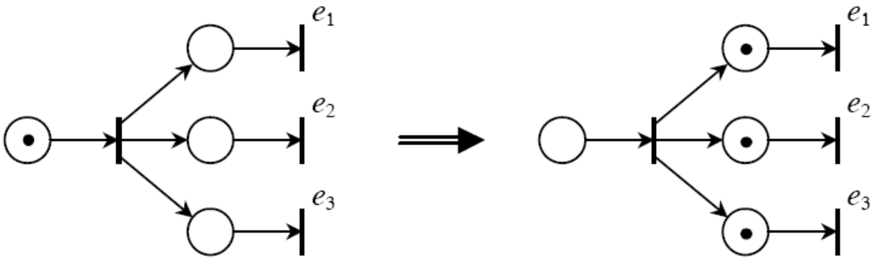
# Sequences

## Sequence of events



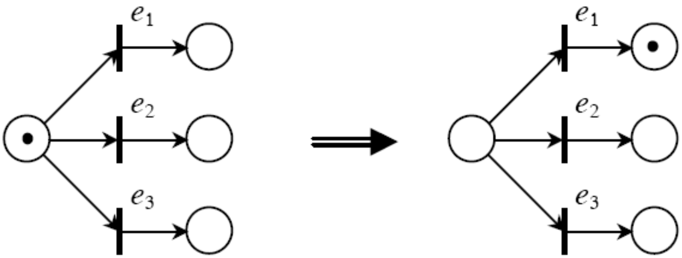
# Parallelism

**More events are simultaneoulsy enabled**



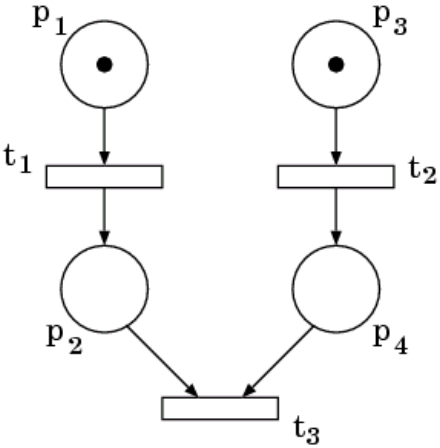
# Choice

**More events are enabled, just one can occur: a kind of non determinism**



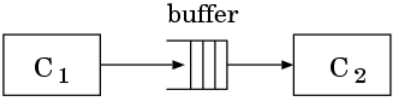
# Synchronization

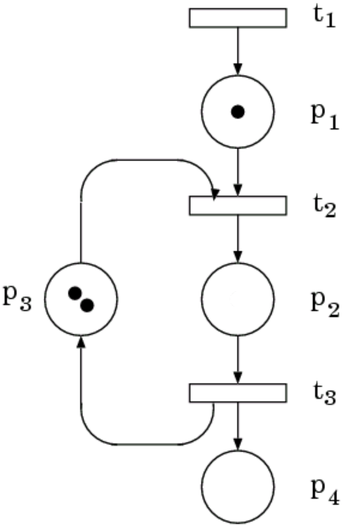
## Synchronization of sequences of events



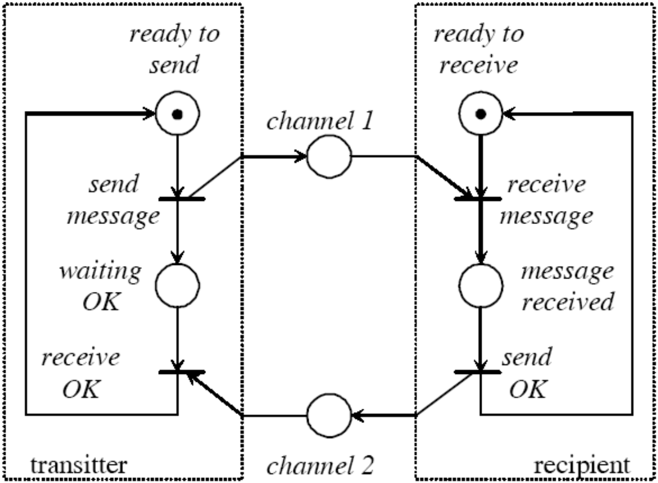
# Limited resources

No more than 2 tokens in $p_2$



# Sender and Receiver model



# Outline

- Definition of Petri nets
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# Timed Petri Nets

For performance and dependability analysis it is necessary to introduce the duration of the events associated with PN transitions.

The structures to extend the PN by time are different.

Delay times can be associated with places, transitions or arcs.
Typically time is associated with timed transitions.

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# Firing rules

*Firing in 3 phases:*

- phase 1- tokens are removed from the input places,
- phase 2- the delay time associated with the transition elapses,
- phase 3- tokens are deposed in the output place.

*Firing in one phase:*

tokens arrive in the input place, enable the transition and stay in the place for the delay time associated with the transition.

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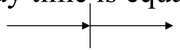
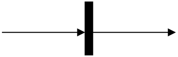
# Classification of Timed Transitions

Immediate transition : transition fires as soon as it is enabled, the associated delay time is equal to zero.

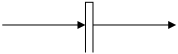


Deterministic timed transitions: the delay associated with the transition is deterministic and usually constant (black box).



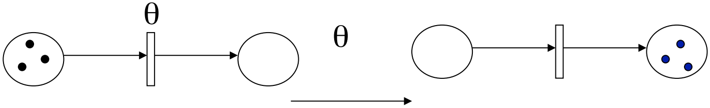
Stochastic timed transition : a stochastic variable is associated to the delay $\theta$ with a known distribution function (white box)

If $\theta$ has exponential distribution $f(\theta)=\lambda e^{-\lambda\theta}$, then the average delay time is $1/\lambda$.

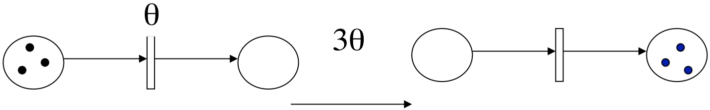


# Server semantic

Infinite server: each transition represents a server that can perform infinite parallel operations.




Single server: each transition represents a server that can perform only one operation at a time.



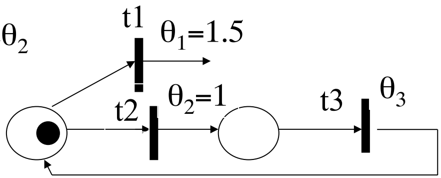

Multiple server: each transition represents a server that can perform k operations at a time.

# Transition memory

Total memory: transition remembers that it has been previously enabled.

Abilitation memory: transition remembers just the actual enabling

$\theta_2<\theta_1<2\theta_2$



# Outline

# Two case studies

- M. P. Fanti, A.M. Mangini, M. Dotoli, W. Ukovich, A Three Level Strategy for the Design and Performance Evaluation of Hospital Departments, IEEE Trans. on Systems, Man, and Cybernetics. Part A, no. 4, vol. 43, July 2013, pp. 742-756.

- M.P. Fanti, A.M. Mangini, G. Pedroncelli, W. Ukovich, Fleet Sizing for Electric Car Sharing Systems in Discrete Event System Frameworks, to appear on the IEEE Transactions on Systems Man and Cybernetics: Systems, 2017.

Basic references:


- *J.L. Peterson, Petri Net Theory and the Modeling of Systems, Prentice Hall, 1981.*

- *T. Murata, "Petri nets: Properties, analysis and applications," Proceedings of the IEEE, Vol. 77, N. 4,p. 541-580, 1989.*