Petri nets: Modeling and Applications

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Outline

- Definition of Petri nets
- Petri net system
- Reachability graph
- Petri net models
- Timed Petri nets
- Applications
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Definitions

Petri Nets (PN) offer advantages because of their twofold representation: graphical and mathematical.

It is a graphical and mathematical tool for the formal description of the logical interactions and the dynamics of complex systems. PN are particularly suited to model:

- Concurrency
- Conflict
- Sequencing
- Synchronization
- Sharing of limited resources
- Mutual exclusion
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The main International Conference: ATPN - Application and Theory of PN Other conferences: Int. IFAC conference, IEEE SMC conference, IEEE CASE, WODES, IEEE CDC etc.
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A Petri net is a bipartite directed graph consisting of two kinds of nodes: places and transitions. Places typically represent conditions within the system being modeled. They are represented by circles.

Transitions represent events occurring in the system that may cause change in the condition of the system. They are represented by bars.

Arcs connect places to transitions and transitions to places (never an arc from a place to a place or from a transition to a transition).
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Input arcs are directed arcs drawn from places to transitions, representing the conditions that need to be satisfied for the event to be activated.

Output arcs are directed arcs drawn from transitions to places, representing the conditions resulting from the occurrence of the event.

Input places of a transition are the set of places that are connected to the transition through input arcs.

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Example

- $p_1$ – resource idle
- $p_2$ – resource busy
- $t_1$ – task arrives
- $t_2$ – task completes
Formally, a Petri net is described by the four-tuple $PN = (P, T, Pre, Post)$ where:

- $P$ is the set of places and $|P| = m$
- $T$ is the set of transitions and $|T| = n$
- $Pre : P \times T \rightarrow N$ is the pre incidence matrix, that specifies the arcs directed from places to transitions
- $Post : P \times T \rightarrow N$ is the post incidence matrix, that specifies the arcs directed from transitions to places
Example

\[ P = \{p_1, p_2\}, \quad |P| = 2 \text{ set of places} \]
\[ T = \{t_1, t_2\}, \quad |T| = 2 \text{ set of transitions} \]
Example

\[
P = \{p_1, p_2, p_3, p_4\}, \quad T = \{t_1, t_2, t_3, t_4, t_5\}
\]

\[
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{array}{l}
p_1 \\
p_2 \\
p_3 \\
p_4 \\
\end{array}

Pre =
\]

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 1 \\
0 & 2 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{array}{l}
p_1 \\
p_2 \\
p_3 \\
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\end{array}

Post =
\]


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Incidence matrix

The incidence matrix is \( C : P \times T \rightarrow \mathbb{Z} \) is defined as \( C = \text{Post} - \text{Pre} \).

\[
C = \begin{bmatrix}
0 & -1 & 0 & 0 & 1 \\
0 & 2 & -1 & -1 & 0 \\
0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 1 & -1
\end{bmatrix}
\]

\( t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \)

\( p_1 \quad p_2 \quad p_3 \quad p_4 \)
Tokens are dots (or integers) associated with places.
A place containing tokens indicates that the corresponding condition holds.
Marking is a function $M : P \rightarrow N$ that associates to each place a non-negative number.
It is a vector listing the number of tokens in each place of the net.
$M_0 = [M_0(p_1)M_0(p_2)M_0(p_3)M_0(p_4)]^T = [1000]^T$
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$M_0$ is the initial marking

$PN = (P, T, Pre, Post)$ is a Petri net

The net system $\langle PN, M_0 \rangle$ is a dynamical system
A transition $t_j \in T$ is enabled at a marking $M$ if and only if for each $p \in \bullet t_j$, it holds $M(p) \geq Pre(p, t_j)$.

Symbol $M[t_j]$ denotes that $t_j \in T$ is enabled at marking $M$. If $M$ does not enable $t_j$, it holds $\neg M[t_j]$. 
Rules of Evolution: enabling condition

\[ M_0 \geq \text{Pre}(\cdot, t_2) \] then transition \( t_2 \) is enabled at \( M_0 \)

An enabled transition may fire (event happens) removing one or more tokens from each input place and depositing one or more tokens in each of its output place.
Rules of Evolution: state equation

When an enabled transition $t_j$ fires, it produces a new marking $M'$:

$$M' = M + \text{Post} (\cdot, t_j) - \text{Pre} (\cdot, t_j) = M + C(\cdot, t_j)$$

or

$$M' = M + C \cdot t_j$$

where $\overrightarrow{t_j}$ is the $n$-dimensional firing vector corresponding to the $j$-th canonical basis vector.

Symbol $M[t_j]M'$ denotes that $t_j \in T$ is enabled, fires and produces marking $M'$. 
Rules of Evolution: state equation

\[ M_0 \geq \text{Pre}(\cdot, t_2) \text{ then transition } t_2 \text{ is enabled at } M_0 \]

\[ t_2 \text{ is enabled and fires...} \]
Rules of Evolution: State equation

Transition \( t_2 \) fires yielding a new marking \( M_0[t_2 > M \) with

\[
M = M_0 - Pre(\cdot, t_2) + Post(\cdot, t_2) = M_0 + C(\cdot, t_2)
\]

\[
\begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}
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A marked net $\langle N, M_0 \rangle$ with $N = (P, T, Pre, Post)$ specifies:

- an initial marking (i.e., state) $M_0$;
- the rules of evolution.

No explicit enumeration of:

- net language, i.e., the set of sequences of transitions that can fire:
  \[
  L(N, M_0) = \{ \sigma \in T^* \mid M_0[\sigma] \};
  \]

- reachability set, i.e., the set of reachable markings:
  \[
  R(N, M_0) = \{ M \in \mathbb{N}^{P} \mid (\exists \sigma \in L(N, M_0)) M_0[\sigma] M \}.
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The information on reachable markings and firing sequences is useful to determine if the net has given properties.
Reachability graph

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Boundedness

Definition

A place \( p \in P \) is \( k \)-bounded if for any marking \( M \in R(N, M_0) \) it holds \( M(p) \leq k \), i.e., in all reachable markings the number of tokens it contains never exceeds \( k \).

Useful to determine maximal capacity or overflow of buffers.

Definition

A marked net \( \langle N, M_0 \rangle \) is \( k \)-bounded if all its places are \( k \)-bounded.

A bounded net has a finite reachability set, while an unbounded net has an infinite reachability set.
Reachability graph

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Liveness

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A transition $t \in T$ is **quasi-live** if there exists a firing sequence $\sigma \in T^*$ such that $M_0[\sigma t]$, i.e., transition $t$ can *eventually* fire.

A transition $t \in T$ is **live** if for any marking $M \in R(N, M_0)$ there exists a firing sequence $\sigma \in T^*$ such that $M[\sigma t]$, i.e., from any reachable marking $t$ can *eventually* fire.

Useful to characterize an event that can occur at least once (quasi-liveness) or that can always eventually occur (liveness).

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Reachability graph

The **reachability graph** of a marked net $\langle N, M_0 \rangle$ is an automaton $G = (X, E, \delta, x_0)$ where:

- $X = R(N, M_0)$, i.e., the states of the automaton are the reachable markings;
- $E = T$, i.e., the events in the alphabet are the transitions of the net;
- for any two reachable markings $M, M'$:

  $$\delta(M, t) = M' \iff M[t] = M',$$

  i.e., there exists an arc labeled $t$ from $M$ to $M'$ on the automaton iff marking $M'$ is reachable from $M$ firing transition $t$;
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Example
Reachability graph

Example

\[
\begin{bmatrix}
1 & 1 & 0 \\
0 & 2 & 0 \\
1 & 0 & 1 \\
2 & 0 & 0 \\
0 & 1 & 1 \\
1 & 2 & 3 \\
2 & 2 & 2 \\
\end{bmatrix}
\]

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Example
Reachability graph

Example

### Reachability Graph

```
1 1 0
0 2 0
1 0 1
2 0 0
0 1 1
```

### Example Petri Net

```
\begin{align*}
p_1 \rightarrow t_1 & \rightarrow p_2 \\
p_2 \rightarrow t_2 & \rightarrow p_1 \\
p_2 \rightarrow t_3 & \rightarrow p_3 \\
p_3 \rightarrow t_1 & \rightarrow p_2 \\
p_2 \rightarrow t_3 & \rightarrow p_1 \\
p_3 \rightarrow t_3 & \rightarrow p_2 \\
\end{align*}
```

\[ \text{Transition Equation: } [1 1 0] \quad \text{Initial Marking: } [0 2 0] \quad \text{Final Marking: } [1 0 1] \]

```
\begin{align*}
t_1 & \rightarrow \text{from } p_1 \\
t_2 & \rightarrow \text{from } p_2 \\
t_1 & \rightarrow \text{from } p_3 \\
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\begin{align*}
t_1 & \rightarrow \text{from } p_2 \\
t_2 & \rightarrow \text{from } p_3 \\
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```

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\[
\begin{pmatrix}
1 & 1 & 0 \\
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1 & 0 & 1 \\
2 & 0 & 0 \\
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Reachability graph

Example
Example:

- $M = [0 \ 1 \ 1]^T$ is reachable
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Sequences

Sequence of events

\[ e_1 \rightarrow e_2 \rightarrow e_3 \]
Parallelism

More events are simultaneously enabled
Choice

More events are enabled, just one can occur: a kind of non determinism
Synchronization

Synchronization of sequences of events
Limited resources

No more than 2 tokens in $p_2$
Sender and Receiver model

Petri net models
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Timed Petri Nets

For performance and dependability analysis it is necessary to introduce the duration of the events associated with PN transitions.

The structures to extend the PN by time are different.

Delay times can be associated with places, transitions or arcs. Typically time is associated with timed transitions.
Timed Petri Nets

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Delay times can be associated with places, transitions or arcs. Typically time is associated with timed transitions.
Firing rules

Firing in 3 phases:
- phase 1- tokens are removed from the input places,
- phase 2- the delay time associated with the transition elapses,
- phase 3- tokens are deposed in the output place.

Firing in one phase:
tokens arrive in the input place, enable the transition and stay in the place for the delay time associated with the transition.
If at the end of the delay time the transition is enabled, then the transition fires.
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Classification of Timed Transitions

**Immediate transition**: transition fires as soon as it is enabled, the associated delay time is equal to zero.

**Deterministic timed transitions**: the delay associated with the transition is deterministic and usually constant (black box).

**Stochastic timed transition**: a stochastic variable is associated to the delay $\theta$ with a known distribution function (white box).

If $\theta$ has exponential distribution $f(\theta)=\lambda e^{-\lambda \theta}$, then the average delay time is $1/\lambda$. 

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Timed Petri nets

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Server semantic

**Infinite server:** each transition represents a server that can perform infinite parallel operations.

<table>
<thead>
<tr>
<th>θ</th>
<th>θ</th>
</tr>
</thead>
</table>

**Single server:** each transition represents a server that can perform only one operation at a time.

<table>
<thead>
<tr>
<th>θ</th>
<th>3θ</th>
</tr>
</thead>
</table>

**Multiple server:** each transition represents a server that can perform k operations at a time.

<table>
<thead>
<tr>
<th>θ</th>
<th>3θ</th>
<th>θ</th>
<th>θ</th>
</tr>
</thead>
</table>
**Transition memory**

**Total memory:** transition remembers that it has been previously enabled.

**Abilitation memory:** transition remembers just the actual enabling

\[
\theta_2 < \theta_1 < 2\theta_2
\]

\[
\theta_1 = 1.5
\]

\[
\theta_2 = 1
\]

\[
\theta_3
\]

\[
t_1 t_2 t_3
\]
Outline

- Definition of Petri nets
- Petri net system
- Reachability graph
- Petri net models
- Timed Petri nets
- Applications
Two case studies


Basic references:
